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Learning Mathematics in an Inclusive and Open Environment: An Interdisciplinary Approach

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Abstract: In this article, we present the first results of the project OPEN-MATH. The research project aims at acknowledging the need for learning environments with differentiation strategies for all. We developed a model for inclusive mathematics learning, based on the Theory of Objectification and a broad idea of differentiation realized through Open Learning. It poses an interdisciplinary research issue that requires the collaboration of two sub-disciplines pertaining to the area of educational studies: Inclusive Education and Mathematics Education. The results we present here are related to the dialogue between theory and practice, whose outcome is a teaching and learning model for inclusion in mathematics. The construction of the teaching and learning model moves along two complementary paths: (1) concerning the theoretical point of view, we implemented connecting theory strategies to network Open Education and the Theory of Objectification; (2) concerning the methodological point of view, we implemented Educational Design Research. The new teaching–learning model is the result of theoretical and methodological validation in real contexts according to an interdisciplinary approach. This study shows the strengths of interdisciplinary research for the pursuit of inclusive mathematics and high standards of learning.

Keywords: theory of objectification; inclusive education; interdisciplinary approach

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1. Introduction

In this paper we would like to present a research topic that lies at the boundaries between Inclusive Education and Mathematics Education. Indeed, from a theoretical and methodological point of view, the encounter of these two educational sciences is the kernel of our study.

We present the first results of OPEN-MATH research project funded by the Free University of Bozen-Bolzano whose aim is to develop inclusive mathematics communities of learners. Inclusion is an ethical and political issue for education in general, but we believe that the nature of mathematical thinking and learning is a breeding ground to scrutinize the effects of inclusive practices and outline the features that bolster and hinders them.

The learning of mathematics plays a prominent role in the educational path of a student. It requires the accomplishment of high cognitive standards, in terms of creativity, rationality, control of several semiotic registers, metacognition etc. Mathematics can be a field of knowledge where the individual's self-esteem and self-efficacy can flourish.

We can also recognize a social and political value in the learning of mathematics, since it is a fundamental tool for contemporary citizens to have access to the complexity of our society. Mathematics is the core of science and technology that are molding the world in unpredictable, unexpected and rapidly changing scenarios. The Italian national curriculum in mathematics [1], attaches significant importance to "mathematics for the citizen" as an important guideline for mathematics teachers. Mathematics can be both an instrument of equity or discrimination according to how and how many students can grasp its cognitive and metacognitive potentials.

The important accomplishments available through mathematics clash against the serious and acknowledged difficulties that research in Mathematics Education has precisely outlined. We mention some of the most important that are strictly related to the ontological and epistemological constitutive traits of mathematics. Firstly, mathematical knowledge refers to ideal entities that do not allow any ostensive reference. The only access to mathematical objects is through signs—both as belonging to complex semiotic systems and as artefacts to carry out mathematical activity—that students need to handle with a specific cognitive competence. In this regard, the main difficulty students are confronted with is to overcome the identification of semiotic representations with the mathematical object they refer to. This is a true obstacle that disguises several pitfalls in the learning of mathematics. Secondly, mathematical concepts require a non-spontaneous cognitive leap from procedural-situated thinking to highly relational-generalized thinking—as Vygotsky [2] would put it—from spontaneous concepts to scientific concepts.

We argue that also the opposite holds true, in that inclusion is a breeding ground for Mathematics Education. Indeed, in the past years there has been a growing interest in foundational issues regarding ethics, equity and the political in Mathematics Education [3–10]. Inclusion is located at the point of intersection of cognition and learning in mathematics, equity, ethics, and the political.

In this regard Ernest [4] (p. 187) argues that,

> "First, the nature of pure of mathematics itself leads to styles of thinking that can be damaging when applied beyond mathematics to social and human issues. Second, the applications of mathematics in society can be deleterious to our humanity unless very carefully monitored and checked. Third, the personal impact of learning mathematics on learners' thinking and life chances can be negative for a minority of less successful students, as well as potentially harmful for successful students."

Radford [11] (p.8) echoes him:

> "This is why a philosophy of mathematics education today appears to me as the space in whose interior an encompassing struggle against the reduction of education in general, and mathematics education in particular, to a technical consumerist view can be organized and deployed. It is in this sense that a philosophy of mathematics education appears as a land of hope—the hope to understand, criticize and *transform* the aims of mathematics education and its concrete practice. This is why I would like to submit that what we need is a *critical and transformative* philosophy of mathematics education (emphasis in original)."

We believe inclusion is a viable path to overcome the risks of mathematics teaching and learning pinpointed by Paul Ernest and contribute to the critical and transformative philosophy advocated by Luis Radford.

In order to make the encounter between Mathematics Education and Inclusive Education fruitful in addressing the issues we mentioned above, it is necessary to go beyond the idea of inclusion as a practice devoted to a specific individual with sociocultural disadvantages or some kind of physical, intellectual or sensorial impairment. In our view, inclusive education should not work to restore a so-called condition of normality; instead, promote the construction of subjectivities in that they react agentically [12] to the cultural-historical environment according to their needs, potentials and difficulties. In the next sections, we will delve into a broad understanding of inclusion that considers the peculiarities of all students, that is termed differentiation [13].

We underline that the aim of our study is the accomplishment of mathematical activities that intertwine the quality of teaching–learning processes in mathematics with the educational philosophy called differentiation. They can be seen as two complementary aspects of the same phenomenon that involve students, teachers and mathematics; the former cannot exist and evolve without the latter and vice versa. We are aiming at configuration of the classroom that becomes:

> "[. . .] as a space of encounters where teachers and students become *presences in the world.* [. . .] the classroom appears as a space of encounters, dissidence, and subversion, where teachers and students become individuals who are more than in the world—they are individuals with a vested interest in one another and in their joint enterprise; individuals who intervene, transform, dream, apprehend, suffer, and hope *together*." [8] (p.5, emphasis in original)

The aim of the paper is not to present the experimental results of the OPEN-MATH project, but rather to describe the theoretical, methodological and practical encounter of Mathematics Education and Inclusive Education, in view of the design of an educational model for inclusive mathematics learning.

In the continuation of the article, we are going to theoretically outline the notion of inclusion in mathematics and a teaching model to pursue it. In Section 2, to we will present a theoretical framework for inclusive mathematics learning and the ensuing teaching model. In Section 3, we will describe the research methodology that informed the implementation and the validation of our teaching model in accomplishing inclusive mathematics learning. In Section 4, we will draw some conclusions and suggest future perspective regarding inclusion and mathematics.

2. Theoretical Framework

2.1. Inclusive Education

Inclusive education has been conceptualized in several different ways. In literature, we find a certain consensus on a general distinction between narrow and broad definition [14–18]. Narrow definitions focus on students with disabilities, their presence in mainstream schools and classes and the needed support. Broad definitions are about school systems and school communities and their commitment and capacity of welcoming all students with all their individual differences, granting participation and effective learning processes. More recent conceptualizations show a clear tendency towards the 'broader' view, focusing on a democratic and quality school for all pupils. Indeed, in 2016 Mel Ainscow argues that inclusion should be understood as a process that aims firstly at the presence of all and everyone at school, but then also at meaningful participation and learning for all [19]. Furthermore, Roger Slee describes inclusion as the process of identifying and overcoming any barriers that hinder some pupils from accessing education and achieving optimal learning and socialization outcomes [20].

Trying to give a more multidimensional nuanced look, Göransson and Nilholm [21] have systematized four different understandings of inclusive education: two refer to a narrow conceptualization in terms of placement of pupils with disabilities in mainstream classrooms and meeting the social and academic needs of pupils with disabilities. The following two instead describe two aspects of the broader conceptualization, that implies on one side meeting the social and academic needs of all pupils and on the other the creation of communities with democratic characteristics.

Against this background, in this research project we choose to work with a broad perspective of inclusive education, focusing on meeting social and academic needs for all pupils and contributing to an equitable and democratic learning community.

2.2. Inclusive Policies

Additionally, on the level of educational policies for inclusion, the trend is that of a shift from a narrow understanding of inclusion to a broader one, as noted by the research of the European Agency on Special Educational Needs [22,23]. At the same time, however, a certain contradictory and confusing co-existence of narrow and broad approaches seems evident, which origins in the fact that in most countries inclusive education policies have their roots in the historical evolution of narrow ones that have rarely led to a comprehensive and coherent rethinking [24–26]. In the Italian context, for example, inconsistencies of this kind are clearly visible. Italy is internationally known for having school legislation that creates ideal conditions for inclusion understood as presence: in fact, since the 1970s, all pupils, regardless of their abilities or disabilities, have the right to attend the school of all and everyone. Nevertheless, special measures and recourses are allocated to students with recognized disabilities and more in general special educational needs. This kind of policy is clearly oriented towards a narrow vision of inclusion, focused on pupils with special educational needs [27]. Only in very recent legislation has attention been explicitly paid to the development of an inclusive school, i.e., one that is able to take into account the differences of all pupils in the name of a learning context that guarantees quality learning and socialization processes for all. In this sense, the broad vision coexists with the narrow vision and this creates some tensions at the level of the development of teaching practices, as we will see in the following paragraphs.

2.3. Differentiation in Teaching and Learning

A broad idea of inclusion poses a great challenge to the way learning processes can be supported in schools both taking into account all students' differences and at the same time granting their participation to a common learning project. Differentiation has been discussed by several authors as a tool that can contribute to tackle the challenge. Spandagou, Grahan and de Bruin [28] have represented the different ways of conceiving it as a continuum that has at one end differentiation understood as a specific strategy provided by a teacher to a pupil or pupils in a class on the basis of their difficulty [29] and at the other end the socio-constructivist view of learning that assumes difference as the norm in learning and that places differentiation in the normality of instructional design for all and all [30]. It is evident here how the pole of differentiation as a specific intervention for a few pupils in difficulty is in line with a "narrow" vision of inclusion, focused on guaranteeing the quality of pathways for pupils with disabilities or special educational needs, while the pole of differentiation for all is aligned to a broad vision of inclusion, sensitive to the individual differences of all children and young people.

The current inconsistency of actual inclusive policies in many European countries described on the basis of the Italian case above, has some consequences on the way differentiation is understood and practiced. In fact, while creating the conditions for differentiation for all in terms of presence through a broad mainstream placement, specific measures for specific pupils, like individual educational plans, suggest the idea that students with certified special educational needs have a right for differentiation, while the same is not granted for all other students. Differentiation is not banned for other students, but at the same time is not required by law. Such a narrow interpretation of differentiation, even if not intentionally, entails some risks. In fact, if differentiation measures become a special tool for students with difficulties, they become that device that marks diversity which risks turning into stigma [31], the stigma of those who are not able to learn like others and therefore do it differently [32,33]. On this background, the project aims at developing a theoretical and methodological framework for inclusive Mathematics Education that rests upon the broad idea of differentiation and offers a contribution in dealing with the tensions arising from inconsistent inclusive policies.

2.4. Open Education

Open Education is defined as an approach that is open for students' autonomous work and for their decisions [32,34,35]. For this reason, for the attention given to every student present in the classroom, we decided, in our project, to implement Open Education as a possible way to actually develop differentiation. Teachers have the role to design a structured learning environment that promotes students' opportunity to organize the learning process for their own, working on different tasks at the same time in the same space. Students are expected to be active in their learning processes, to be aware about the way they learn and to take decisions according to that.

Historically, the approach of Open Education found its roots in a movement [36], which in the 1960s and 1970s in Germany brought together different didactic reflections and applications united by: (1) a definition of the role of the learner as capable of making choices for their own learning path and autonomous in the planning and carrying out of activities; (2) a definition of the role of the teacher as an organizer of contexts capable of valuing the interests, needs and abilities of each pupil and offering support for their planning and, finally; (3) a definition of learning based on doing and discovery in which everyone is responsible for his/her own learning path [32,37].

Falko Peschel [35], defines the concept of openness more precisely, declining four categories that describe three possible areas of decision-making for pupils in terms of learning: organization (spaces, times, learning partners), methodology (how to solve a task), aims and objectives (content and goals). A fourth category describes the category of openness of relationships and rules and refers to community rules constructed in a democratic and participatory way, in an educational style that becomes a way of being together and where the choices of individuals must be measured against the choices of others and of the group. According to Falko Peschel, education is open if all four categories of openness are realized, thus imagining a very radical model of openness.

Bohl and Kuckartz [34] take up Peschel's categorization of openness but use it to recognize the value of less radical forms of openness. The activation of certain strategies that allow freedom of choice in even just one of the four categories described cannot be called Open Education in the full sense, but it has all the legitimacy of a form of educational openness that moves in the direction of recognizing pupils' autonomy and self-determination.

In relation to a broad idea of differentiation, Open Learning has a great potential for at least two reasons. Firstly, the pupil-centered perspective leads to a focus on learning instead of teaching and encourages the development of class settings where learning processes are decentralized and plural: several students working on several tasks and each of them is going through his/her own valuable learning process. Secondly, the fact that students are expected to take decisions, introduces a sort of "self-determined differentiation": in a learning landscape that offers plural learning opportunities, teachers do not always need to match pupils and learning task, instead they support pupils' autonomous and competent choice [32].

Specifically, the stations [30,32]—one of the instructional strategies referring to the general approach of Open Learning—represent, as we stated before in this work, a way to put into practice differentiation according to the two elements described above. Briefly, stations are different learning activities connected to one main topic are structured in different stations. Stations allow different learners to access the same topic/competence in different ways, by means of different contents and methods. Each station is built up of the materials and instruction necessary for the planned activity with the intention to support learners' autonomous work. Students learn moving from one station to the other. Not all students do the same stations: in the same learning landscape, each student follows his/her own learning path. The choice of the individual leach learners' stations can be done both by teachers and by pupils. The stations give the opportunity to work on joint topic and at the same time to take into consideration different learning ways and preferences, different interests or different competence levels.

2.5. Transmissive, Progressive and Socio-Cultural Approaches in Mathematics Education

Radford [7,8], drawing on dialectic materialistic understanding of thinking and learning, introduces the notion of joint labour that envisages teaching and learning as the same activity where students and teacher engage together in the common labor, with their specific roles and contributions to the production of knowledge. Knowledge is no more a commodity exchanged between the teacher and the student, but it emerges from a variety of mathematical practices shared in the classroom: problem solving, communication, discussion groups, communities of inquiry etc.

Within sociocultural approaches, the Theory of Objectification is arguably one of the most robust and acknowledged theorizations of mathematical teaching and learning. Rooted in Leont'ev's [38,39] Activity Theory paradigm, the theory goes beyond rationalist or individualist views of cognition, reconciling the subjective and the objective, the sensual and the conceptual, the ideal and material. Cognition, revisited from a non-mentalist standpoint, is conceived as a sociocultural and historical practice, namely a praxis cogitans. Radford points out that "thinking is considered to be a mediated reflection in accordance with the *form or mode of the activity of individuals*" [5] (p. 218, emphasis in original). Conceptual objects, thinking, learning and meaning in mathematics are intertwined in reflexive mediated activity.

Learning is a specific praxis cogitans that Radford [5] terms a process of objectification. In its etymological meaning it refers to the process that allows the student to notice, find and encounter the cultural object. The artifacts that mediate reflexive activity and accomplish the objectification processes are called semiotic means of objectification [40] and they cover the whole range of possible ideal and material resources. In Radford's [40] (p. 41, emphasis in original) words:

> "These objects, tools, linguistic devices, and signs that individuals intentionally use in social meaning-making processes to achieve a stable form of awareness, to make apparent their intentions, and to carry out their actions to attain the goal of their activities, I call *semiotic means of objectification*."

The Theory of Objectification can be set into the strand of embodied cognition in mathematics—for an overview we refer the reader to Radford et al. [41]. Radford [7,12], resorting to a dialectic materialistic stance, conceives embodiment as a sensuous cognition, that is, a multimodal sentient form of responding to the world sprouting from cultural and historical activity. Cultural and historical activity intertwines, in sensuous cognition, senses, feelings, materiality, and the conceptual realm. The multimodality of the individual's response intertwines the manifold possibilities of human perceptions (sight, touch, sensorimotor activity, feelings) with the modes of activity realized by the variety of semiotic means of objectification that cover the whole spectrum of human experiences, ideal and material, sensual and conceptual, subjective and objective.

The materiality of cognition is not something subsumed in the mind to acquire the nature of a concept, but the material is consubstantial to the conceptual. Senses, feelings, materiality and the conceptual realm culturally and socially develop into what [7] terms "highly sensitive cultural objects—*theoreticians*" [7] (p. 353, emphasis in original), in which the material and the ideal are tuned into the objectification of mathematical generality. The multimodal nature of sensuous cognition allows us to outline levels of generality [7] at which the student objectifies the mathematical concept. The level of generality specifies the blending of ideal and material in the process of objectification, according to the artefacts that realize the process of objectification:

- *Factual generalization*—characterized by perception, feelings, movement, spatial and temporal elements of the students' physical environment—is accounted for mainly by gestures, bodily movements, material objects and deictic and generative use of natural language.
- *Contextual generalization* intertwines material perception, movement and feelings with a new perceptual field in which emergent objects are detached from mediated sensory perception. Students start introducing more ideal semiotic means of objectification, such as new linguistic terms, natural language and the first elements of symbolic language.
- In *symbolic generalizations*, perception is no longer embedded or related to a concrete space-time context but in a new abstract and relational "space" where mathematical activity is carried out mainly by symbolic language.

The dialectic interplay between a cultural-historical environment, the individual and reflexive activity gives rise to a double-sided construct: objectification-subjectification. We described objectification in the previous paragraphs. Subjectification [12], the counterpart of objectification, is related to the production of subjectivities as they engage in the reflexive mediated activity. If objectification pertains to the process of *knowing*, subjectification pertains to the process of becoming, that is, the changes and development of the individual. The Theory of Objectification outlines a dialectical co-production between individuals and their cultural and historical reality. Radford [12] (p. 43) conceives the individual as:

> "[...] an entity in flux, in perpetual becoming—an entity who, through practical activity (like play) is continuously inscribing herself in the social world and, in doing so, she is continuously produced and co-producing herself within the limits and possibilities of her culture."

2.6. Networking the Theory of Objectification and Open Learning

In view of providing a theoretical stance towards inclusive mathematical practices, we propose an encounter of the Theory of Objectification with Open Education according to the Networking Theories paradigm [42]. For an overview on theories and connecting theories in Mathematics Education we refer the reader to [43–48].

The outcome of the networking strategy should encompass the following features connected to inclusion as differentiation:

- Outline the notion of inclusion in mathematics.
- Provide learning activities that meet personal needs, potentials and talents of each student.
- Nurture both the individual's distinctive traits and social interaction.
- Outline a teaching–learning model to be implemented in everyday mathematics classroom, which in our study involves grade seven students.

We carry out the connection between the Theory of Objectification and Open Learning within the Networking Theories paradigm developed in the Mathematics Education community. In particular, we refer to the connecting strategies developed by Prediger, Bikner-Ahsbahs, and Arzarello [49] and Prediger and Bikner-Ahsbahs [42]. The aim of connecting theory is to grasp the richness of the diversity of theoretical perspectives to enhance communication between different viewpoints, integration of empirical results, and the recognition of strengths and weaknesses among theories and scientific progress.

Prediger, Bikner-Ahsbahs and Arzarello [49] propose a "landscape" of possible connecting strategies that balance the plots of identity and integration. The following schema (Figure 1) adapted from the article shows the networking strategies ordered according to the possible blends of integration and differentiation.

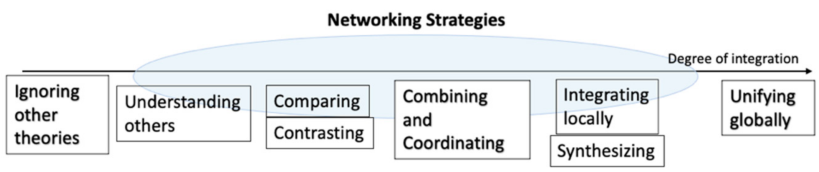


Figure 1. The "landscape" of possible connecting strategies. Adapted from [49] (p. 170).

At the opposite extremes of the "landscape" lie ignoring other theories and unifying globally, which are not considered since they do not incorporate both the plots of integration and differentiation, which characterize the semiosphere [48], a social-cultural space where theories develop and coexist as multicultural identities. We will focus on the strategy combining and coordinating.

Prediger and Bikner-Ahsbahs [42] (pp. 119–120, emphasis in original) argue that:

> "Following the idea of triangulation, combining and coordinating means looking at the same phenomenon from different theoretical perspectives as a method for deepening insights into the phenomenon. [. . .] *Combining* theoretical approaches does not necessitate the complete compatibility of the theoretical approaches under consideration. Even theories with conflicting basic assumptions can be combined in order to get a multi-faceted insight into an empirical phenomenon in view. In contrast, we use the word *coordinating* when a conceptual framework [. . .] is built by fitting together elements from different theories for making sense

of an empirical phenomenon. A conceptual framework is not a new theoretical approach but a pragmatic bricolage for the purpose of understanding empirical phenomena."

We remark that the connecting theories paradigm has been developed in Mathematics Education research. Nevertheless, we believe that the connecting strategies devised by Prediger, Bikner-Ahsbahs, and Arzarello [49] apply in a broadened understanding of the semiosphere as the sociocultural space of educational sciences.

The aim of our study is creating a conceptual framework for inclusive mathematical teaching–learning activities. The Theory of Objectification and the Open Learning Approach are not completely compatible in the enlarged semiosphere of educational sciences. The former rests on sociocultural underpinnings centered on joint labor and being with others, whereas the latter on socio-constructivist underpinnings that stress the role of autonomy and self-determination. Social interaction and the individual's agency are present in both theories but with a different hierarchical position in their system of principles. They are not conflicting theories, and they can be *combined* in order to get a multi-faceted insight into inclusive mathematical practices. The counterpart of combining, in this connecting strategy, is coordinating that allows us to fit together elements from the two theories in view of a conceptual framework for inclusion in mathematics. We cannot go beyond the combining-coordinating degree of integration because we would overcome their boundaries, which set the limits of discourse of a theory, beyond which both theories would start contradicting themselves. Therefore, we would discard the balance between the plots of integration and identity.

2.7. A Conceptual Framework for Inclusion in Mathematics

The combining-coordinating strategy provides the basic bricks that amount to a conceptual framework for inclusion in mathematics. At the core of our inclusive frameworks lies the dialectics between two facets: social interaction and individual self-determination.

Combining the Theory of Objectification and Open learning allows us to gather insight into the two features that contribute to our framework. Regarding social interaction, the Theory of Objectification precisely frames shared mathematical practices in terms of reflexive mediated activity, objectification-subjectification processes and semiotic means of objectification against the cultural-historical background. Regarding individual self-determination, Open Learning and the inclusion paradigm as differentiation for all provide insight into opening learning in time and space, the use of materials, self-efficacy, levels of participation, students' wellbeing and self-determination.

Coordinating the two theories allows for the construction of a conceptual frame for inclusive mathematics made up of the following components that comply to the dialectics between social interaction and individual self-determination (Figure 2):

- *Definition of inclusion*. Inclusion is conceived as the dialectical and critical positioning of all students in the cultural-historical practice of mathematics, who act, feel and think according to their individual distinctive traits to pursue their project of life.
- *Mathematical activity.* Mathematical reflexive mediated activity, in its multimodal acceptation, is the meeting point of the social and individual dimension of mathematical learning. The notion of sensuous cognition allows us to develop distinctively the plots of social interaction and individual self-determination, although social interaction and individual self-determination are inseparable in their dialectical interplay. Semiotics means of objectification allow multimodal activity both as open learning and joint labor.
- *Teaching-learning model.* Starting from the Activity Theory methodology [50] we have developed an inclusive lesson plan that intertwines social interaction and individual self-determination. We have inserted in the original Activity Theory design elements of Open Learning and broadened the original schema with elements borrowed from Pfeiffer and Jones' [51] experiential learning cycle. The outcome is a new learning cycle for inclusion called Open Activity Theory Lesson Plan (OATLP).

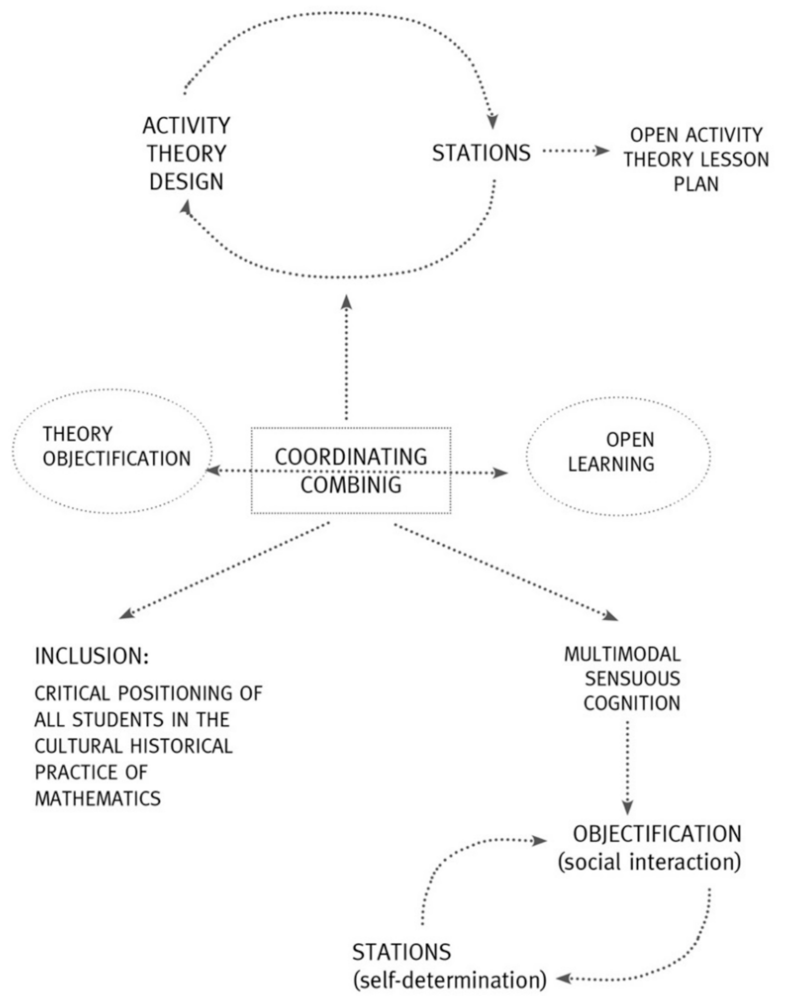


Figure 2. Schema of "combining-coordinating" conceptual framework.

We stress that, according to our dynamic understanding of theory, our conceptual framework is the outcome of the profound dialogue between theory and practice carried out during the OPEN-MATH project. In the next section, we will describe how such a dialogue was carried out during the project. Furthermore, we are moving along a pragmatic bricolage for the purpose of understanding and implementing inclusive mathematics learning as an empirical phenomenon.

We conclude this section providing further insights about the components of our framework.

As regards the notion of inclusion, we have outlined two sets of categories that allow us to recognize the accomplishment of inclusion in mathematics. The first set regards sensuous cognition and the positioning of the individual as a new voice and presence in the mathematics reflexive mediated activity:

- Use of semiotic means of objectification according to the distinctive traits of each student.
- Use of semiotic means of objectification as drivers of the student's positioning in the mathematical activity as an outcome of the interplay between social interaction and open learning activities.
- The student's movement across factual, contextual and symbolic generalizations.

The second set refers to how each student takes part in mathematical activity according to the following levels of participation:

- Level −1: exclusion.
- Level 0: absence of the student.
- Level 1: presence without interaction.

- Level 2: interaction with first level communicative interpersonal skills.
- Level 3: interaction with second level communicative interpersonal skills.
- Level 4: interaction with leadership skills (goal oriented/climate oriented).

We conclude focusing on the Open Activity Theory Lesson Plan. It is the pragmatic realization of the dialectics between social interaction and individual self-determination. The cycle was originally prompted by the Activity Theory design (Figure 3):

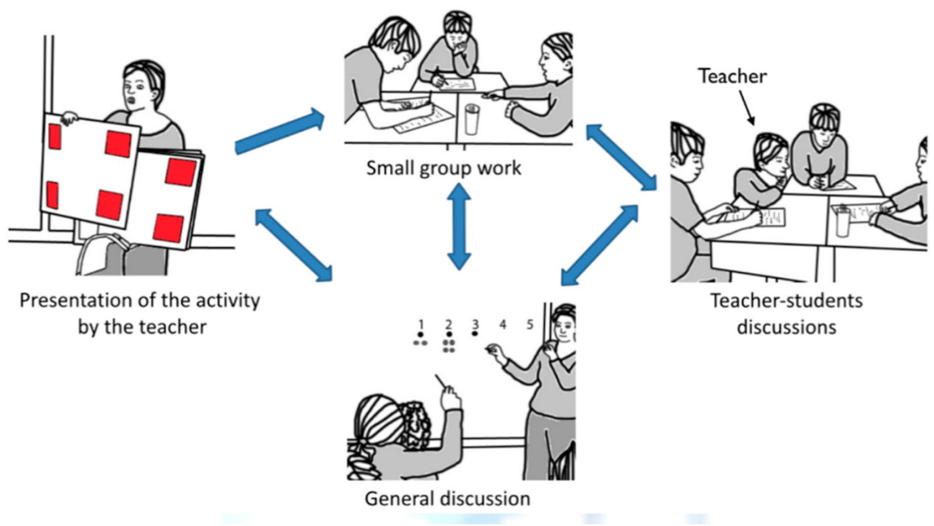


Figure 3. The Activity Theory design [50] (p. 556).

In order to foster the students' personal implication and allow the design to develop along several cycles in a suitable time span for the teaching–learning of a didactical unit, we broadened the schema according to Pfeiffer and Jones [51] experimental learning cycle. Furthermore, we have inserted Open Learning activities (stations and differentiated work) in strategic turning points of the cycle, thus providing the students with the skills to participate in the mathematical activity, according to their distinctive traits.

After a trial-and-error approach with the students, described in Section 3, we eventually arrived at the following design for the Open Learning Activity Theory Lesson Plan (Figure 4):

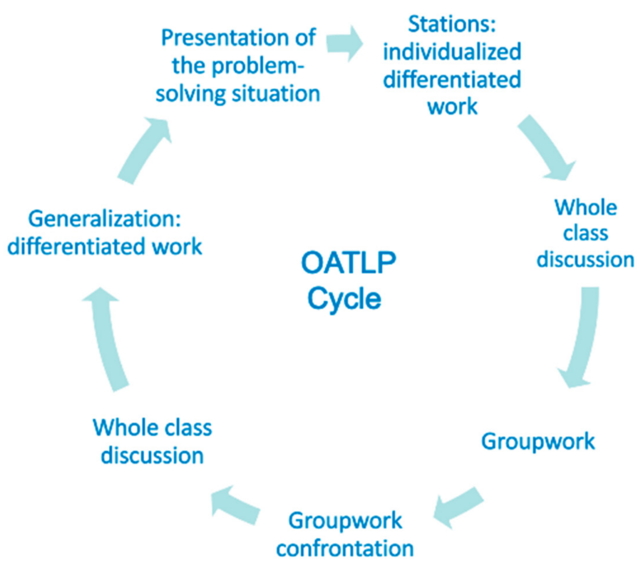


Figure 4. The structure of the OATLP cycle.

In the following section we will delve into more practical details regarding the functioning of the cycle and the research methodology that led to this teaching learning model.

3. Methodology

The project OPEN-MATH follows the methodology of Educational Design Research (EDR), known also as design-based research. In this section we outline the characteristics of EDR, its aims and functioning, and in Section 3.3 we show how our research is put into practice for the construction of the OATLP model along this methodology.

3.1. Educational Design Research

Our research methodology, based on EDR, has been informed by the combining-coordinating networking strategy. Generally speaking, the aim of educational design research relates to the enhancement of theoretical understanding, along the design of effective learning intervention. These two goals in educational research are deeply intertwined, even if historically there is a separation between educational designers and researcher.

EDR constitutes the methodological underpinning of the project Open Math: within the context of EDR a strong reciprocity between theoretical reflection and the development of educational interventions is promoted. Indeed, theoretical understanding in EDR should [52] support the design, frame the scientific inquiry, be abstracted from empirical findings and advance along the intervention and its testing. The pragmatic aim of EDR, that of finding solution to practical problems, is then realized with interventions implemented and tested as inputs to educational environments in real context.

Characteristics of EDR that can be found in literature are different [53–56], but there is a general agreement on some salient features [52]. EDR is defined as:

- *Theoretically oriented*, in the sense that it uses existent theories to define inquiries, that in the end will help to foster theory development. In the case of OPEN-MATH, the theory of objectification and the open learning approach are combined.
- *The theoretical understanding* is used to design solutions to real problems in real contexts, and to understand how and why the design functions, and in which aspects: addressing middle school's differences of access to significant mathematics activities, in the case of this project.
- *Interventionist*, aims at changing the existing situation, impacting positively on practice: the OATLP cycle is evaluated in order to find ways to manage the complexity of the classroom in order to reach significant learning for each student.
- *Collaborative*, which means that it is conducted collaborating with actors connected to the problem that we want to solve: in the case of OPEN-MATH, researchers actively collaborated with students and teachers, who by means of feedbacks showed the direction for changes in the design and for adaptation of the research.
- *Responsively grounded in data*: his products are shaped by participants, by literature and by the field, it is conducted to explore the complex realities of teaching and learning context.
- *Iterative*: is made of cycles of studies that repeat themselves, developing, testing and refining both design and hypothesis, as it is the case of the repetition and continuous adjustments of the OATLP cycle over one school year.

The methods for conducting EDR are also various, and usually more than one data collection is performed in order to understand a single phenomenon, combining different methodologies. Anyway, a generic model for conducting educational design research can be described following McKenney and Reeves [52]. A generic model for conducting EDR, is schematized in Figure 5.

The three squares represent the three main phases of EDR: *analysis and exploration*, *design and construction* and *evaluation and reflection*. Each phase has a double name, the first name relates to the practical outcome of the research, the educational intervention, the second one relates to the theoretical aim. The two aims are indicated in the two rectangles on the right. The arrows between elements indicate the iterativity and flexibility of the process along implementation and spread, that grow during the research process in an interaction with practice that become more significant along the different phases.

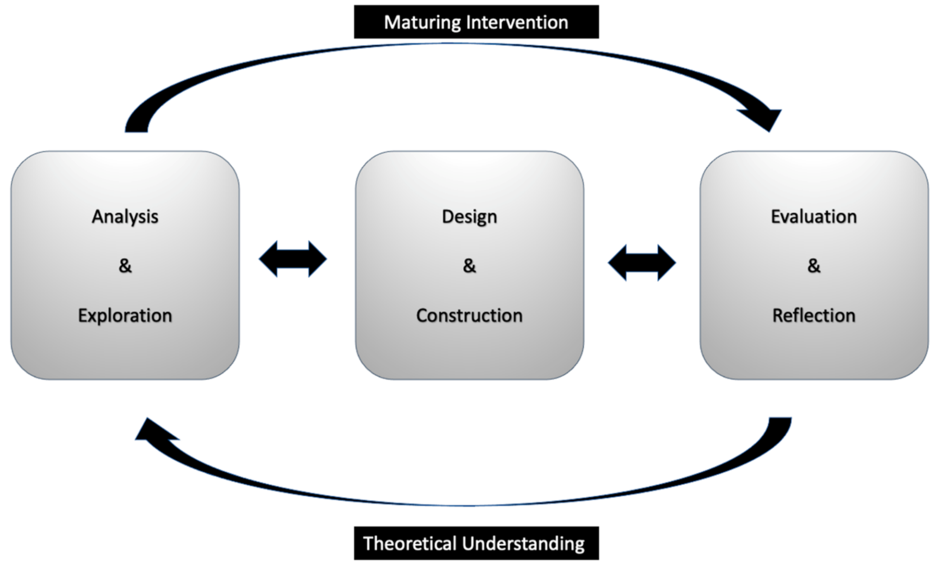


Figure 5. A model for conducting EDR. Elaboration from [52].

3.2. Features of the Methodology of the Project OPEN-MATH as an EDR

In this section we show the structure of OPEN-MATH project, making explicit its connection with EDR model. To start, a schema regarding the research is shown in Table 1, where the double aim, pragmatic and theoretical, of the research is highlighted as *intervention developed* and *knowledge created*.

Table 1. Schema of the research.

Problem	**Middle School's Differences of Access to Significant Mathematics Activities**
Main Focus	Developing models to foster inclusion in the mathematics class
Intervention developed	Open Activity Theory Lesson Plan
Knowledge created	Complementary interdisciplinary approach on inclusion in the mathematics class
Research methods used	Observations Interviews Document analyses Six qualitative case studies

More specifically, the main data of the Open Math project come from a series of five interventions with a grade seven class along one school year: each intervention follows the OATLP structure and lasts between four and nine mathematics hours (the change of duration is due to the process of adaptation of the design) and focus on one mathematics topic that is faced by the student during the school year. An additional intervention in school of the duration of 4 h has been made at the beginning of the project with the aim to know the students. Within this intervention some activities were conducted where a description of students' perceived learning style and learning preferences was asked.

The class is composed by 17 students, the mathematics teachers, and in some of the mathematics hours by a support teacher, assigned to the class because of the presence of a student with an intellectual disability. The mathematics teacher participates in the design of the specific task of every iteration, choosing the topic according to what the class is working on at the moment and modifying the proposed tasks when needed. The teacher conducts the intervention with his classroom and videorecords one group during groupwork and whole class discussions. The teacher collects all students' protocols for the researchers' analysis. Due to pandemic restrictions, no researcher was allowed in the classroom during the intervention. Only the first two weeks of intervention, taking place in October 2020, saw the presence of three researchers: presence that was fundamental to introduce the project to the students and getting to know them. It was also fundamental to see the distribution of the students in the different groups and to observe the whole class at work. After the first cycle, due to the aggravation of the COVID-19 pandemic, the teacher had to start the autonomous videorecording of one group at every iteration of the cycle. The researchers conduct online interviews after every cycle of activities with six students chosen as case studies.

In summary, the data available to the researchers are:

- *Videorecording of the groupwork*: the groups are four in total, three composed by four students and one of five students, and at every intervention the teacher records a different group.
- Some *recordings* of whole class discussions.
- *Student's interviews*: six students chosen as case studies, and interviewed after every cycle of intervention. The chosen students have different mathematical abilities, belong to different groups, and have different preferences regarding individual or group work.
- A *teacher interview* was conducted after every cycle of intervention.
- *Student protocols* came from stations and from groupwork.

We speak more consistently about data related to one specific implementation of OATLP in Section 3.3, here we want to focus more specifically on aspects of the research connected with the modus operandi of EDR. The project OPEN-MATH embodies all the characteristics of EDR indicated above along specific directions, that are specified here, recalling the name of the single characteristics previously listed.

Theoretical Orientation and Pragmatic Aim

The orientation of the project has already been described as both theoretically and pragmatically oriented, here we highlight the structure of the research, giving a more detailed look at the argumentative structure of our claims, separating the claims related to the design from the claims related to the theory. In order to do so, we follow the Sandoval approach [57] related to EDR, making explicit the conjecture map of the research and motivating its various assumptions (Figure 6).

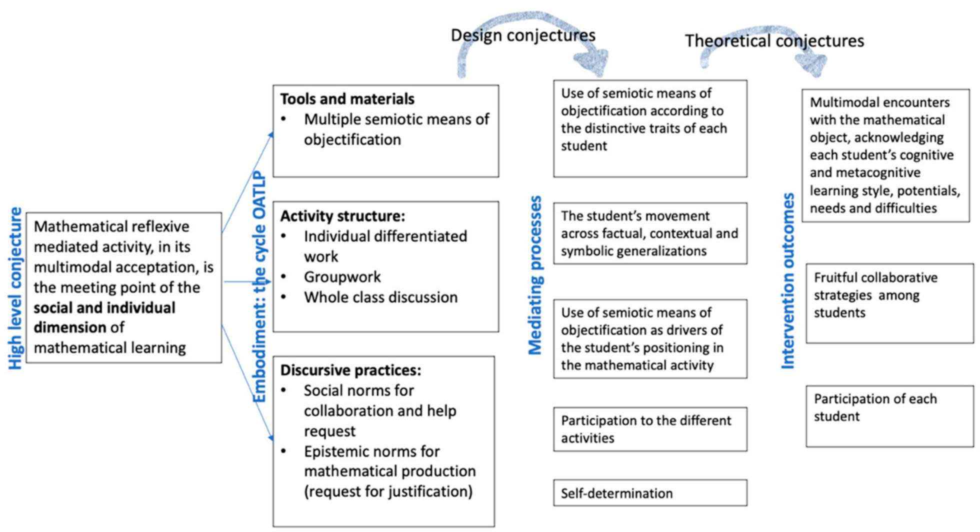


Figure 6. Conjecture mapping of the project OPEN-MATH.

The conjecture map is itself a research tool, so it will be modified to adapt to new perspectives and hypotheses made during the research work. According to Sandoval [57] it is necessary to "move beyond reflections about the kinds of knowledge design research can produce to develop systematic approaches to the conduct of design research" [57] (p. 19), and conjecture mapping is an endeavor in that direction. In order to approach EDR systematically, it is fundamental to separate conjectures related to the design from conjectures related to theories of learning and theoretical construct involved in the research. Nonetheless, it is necessary to make explicit how these two kinds of conjectures are expected to be related in the production of the outcome. The main assumption behind the map itself is that learning environments embody hypotheses about learning, about its aims and its functioning, that are implicit in the researcher's view but that must be made explicit in order to study an educational intervention.

The map must be read from left to right: generally speaking, a research project, as does the project OPEN-MATH, starts from a general hypothesis that drives the work. In this case, the general theoretical framework, related to Theory of Objectification and Open Didactics, has been delineated in Section 2.7, but the starting point of research is the assumption that reflexive mediated activity, in its multimodal acceptation, is the meeting point of the social and individual dimension of mathematical learning. As stated in Section 1, according to the Theory of Objectification, mathematical knowledge is objectified in reflexive activities mediated by semiotic means of objectification. They allow multimodal encounters with the mathematical object, acknowledging each student's cognitive and metacognitive learning style, potentials, needs and difficulties. A structural change in the activity design used within the Theory of Objectification, which allows access to groupwork for each student of the class, is at the basis of the project. Our hypothesis is that didactic differentiation implemented via Open Learning (stations) in individual work fosters the positioning of each student in the mathematical practice, accomplishing both participation in class activities and high standards of mathematics learning.

The high-level conjecture has driven the first definition of the OATLP cycle, that embodies this assumption, organizing the activities in different phases alternating individual, group and whole class work, according to indications made explicit in the second column of the map.

The design conjectures of the project, that is, the conjectures that we did regarding the design of OATLP and his functioning, are that it fosters some mediating processes related to inclusion in the mathematics class. The mediating processes hypothesized are in the third column of the map.

The listed mediating processes enable significant learning for each student, as well as inclusion, consistently with the theoretical framework we refer to. The outcomes of this kind of learning are in the fourth column, and the implication between the third and the fourth columns is underpinned by the theoretical conjectures consistent with our conceptual framework. For example, multimodal encounters with the mathematical object in the fourth column is theoretically justified by learning as a sensuous cognition, which is translated in the mediating processes in the third column.

The map as it is presented allowed us to implement the combining-coordinating networking strategy, defining what aspects of the project are related to each theory in use, and their relationship with the design of the intervention. It contributes also to specifying how the theoretical assumption works synergically with the design assumption in order to develop an effective didactical model.

3.3. Implementation of the EDR Methodology

In this section, we describe the use of the EDR schema in the construction and refining of the Open Math Activity Theory Lesson Plan, whose main features are represented in the conjecture map (Figure 6).

The macro-phases of EDR have been developed as follows:

1. *Analysis and exploration:* Definition of the theoretical framework and of the first structure of the cycle OATLP.
2. *Design and construction:* practical implementation of the OATLP cycle in constant dialogue between theory and practice. More specifically, the first four implementations

of the cycle (the fourth has yet to be designed) involved the following *micro-phases* performed according to the EDR schema:

a. *October: Ratios and proportions.* This implementation led to a redefinition of the stations and of their design: more variability among the different stations, specific focus on the mathematical object, multimodality and sensuous cognition. We also decided to make explicit how to manage help requests and group roles. Definition of the elements of the cycle, shifting from the original Activity Theory design with the insertion of stations into Pfeiffer and Jones' Experiential Cycle adapted to our conceptual framework and overall inclusive objective.
b. *November: Circle and circumference.* This implementation highlighted the need for longer time in relation to a single cycle and the insertion of a new phase devoted to whole class discussion after the stations.
c. *January: Pythagorean Theorem.* Implementation of the actual structure of the OATLP cycle (Figure 4)
d. March: Geometry problems, in progress

3. *Evaluation and reflection*: This macro-phase was not part of the OPEN-MATH project, but it will be developed in further research.

Here we describe in detail the implementation of the OATLP related to the Pythagorean theorem in its final form that takes into account the feedbacks from implementation of the previous micro-phases. The time schedule of this cycle has been:

- 0.5 h presentation of the problem-solving situation,
- 3.5 h stations,
- 0.5 h whole class discussion,
- 1.5 h groupwork,
- 0.5 h group confrontation,
- 0.5 h whole class discussion,
- 2 h generalization: groupwork and whole class discussion.

The students during the stations had the possibility to explore the statement of the theorem and its meaning from six different perspectives and through different semiotic means of objectification (Figure 7).

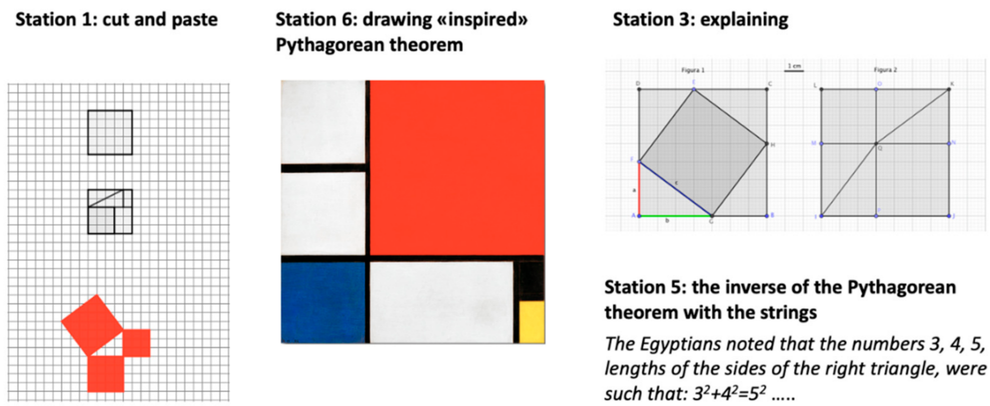
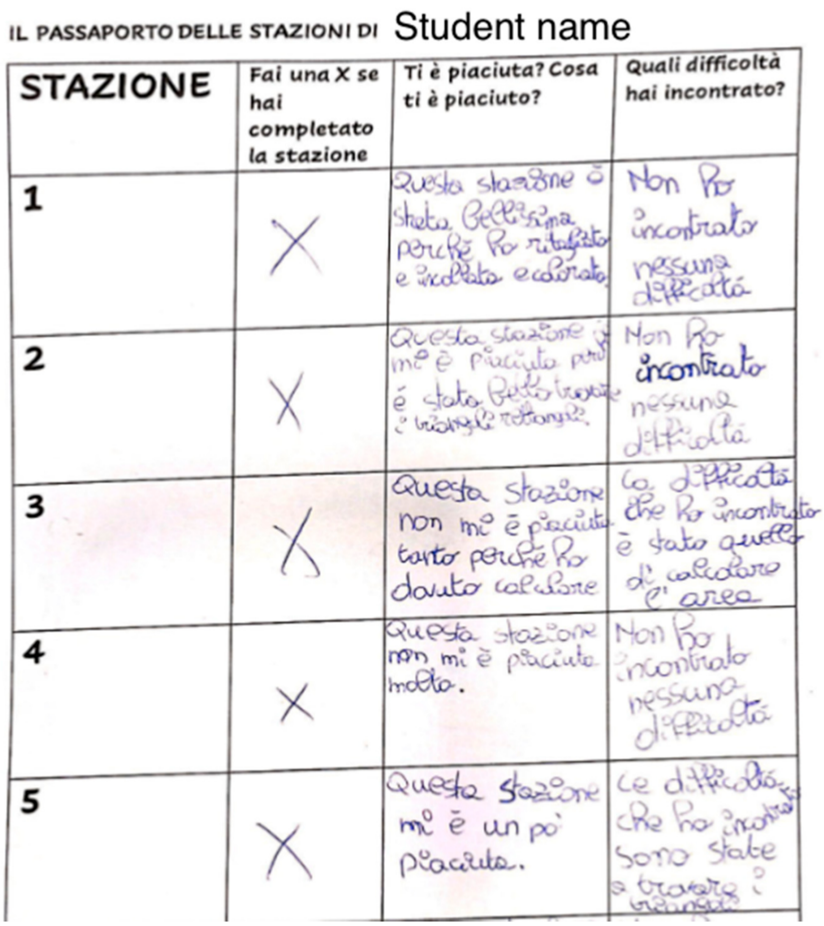


Figure 7. Examples from the stations.

Each student at this stage of the work has all the worksheets, and any materials available and is put in the position to choose which stations to tackle, how many, and in what order. Stations consider different learning styles, encouraging more manual-embodied (station 1), more analytical-ideal (station 3), or divergent (station 6) approaches (Figure 8), or combining several approaches into one activity (station 5). A passport is available to each student (Figure 8), in which he/she are asked to fill in what stations he/she completed, what difficulties he or she encountered, and the degree of appreciation, in order to track each person's work. Free space for suggestions and comment is left at the bottom of the passport. From the second implementation of OATLP we have implemented explicit strategies students could use to ask for help during the station activity. This helps the students to reflect on their management of the activities, and the teacher in providing them support. The possibilities they can choose from, according to open learning are: three cards of different colors are given to each student; white indicates that the student is working autonomously, green that he or she needs help from a classmate and blue that he or she needs aid from the teacher. The chosen card must be visible on the student's desk. A reflection about the meaning of help and its modalities is conducted during a group discussion in relation to mathematical activities.



IL PASSAPORTO DELLE STAZIONI DI Student name

STAZIONE	Fai una X se hai completato la stazione	Ti è piaciuta? Cosa ti è piaciuto?	Quali difficoltà hai incontrato?
1	X	Questa stazione è stata bellissima perché ho ritagliato e incollato e colorato	Non ho incontrato nessuna difficoltà
2	X	Questa stazione mi è piaciuta perché è stato bello trovare il triangolo rettangolo	Non ho incontrato nessuna difficoltà
3	X	Questa stazione non mi è piaciuta tanto perché ho dovuto calcolare	La difficoltà che ho incontrato è stata quella di calcolare l'area
4	X	Questa stazione non mi è piaciuta molto.	Non ho incontrato nessuna difficoltà
5	X	Questa stazione mi è un po' piaciuta.	Le difficoltà che ho incontrato sono state a trovare i [illegible]

Figure 8. An example of the student's passport. The student marks with an X the completed stations in the second column, in the third column, he/she expresses like or dislike towards a specific activity and in the fourth if he or she highlights encountered difficulties.

After the second phase (station activities), the students discuss the difficulties they encountered with the teacher and compare their individual perspective on the formulation of the theorem (whole class discussion). Then, they are divided into groups and given the materials for the groupwork. In the group every student has a role, chosen among four different roles that are fixed from the first iteration of OATLP: the designer, who writes the outcome of the work to be presented, the verbalist, who writes what happens in the group, what problems emerge, and what difficulties arise during work, a mediator, who checks that everybody in the group has the possibility to participate and is understanding, and a time controller, who checks the timing and helps the group to focus on the activities.

During groupwork, the students are initially asked to solve a problem of application of the theorem (Figure 9) and justify their solution, then the group must invent a problem related to the Pythagorean theorem to be solved by another group.

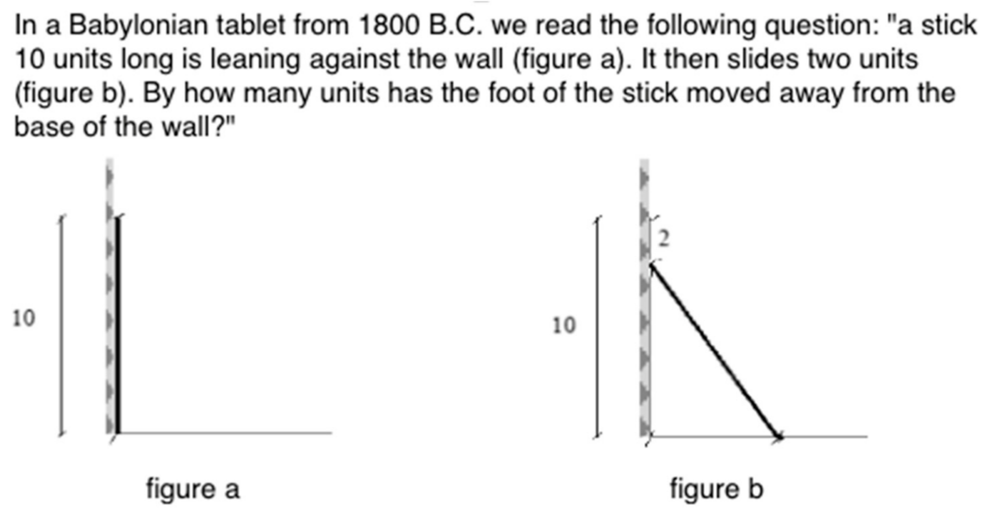


Figure 9. The starting problem of the groupwork, inspired from question C9 of the national assessment test INVALSI 2008 for grade 8.

The problems are then exchanged (group confrontation) and finally a collective discussion (whole class discussion) is led by the teacher on the different methods of solving and constructing the problems. After the discussion, students are asked to work out a new problem (generalization) associated with Pythagoras to share with classmates. The students work in groups again.

In this section, we do not dwell into the analysis of data related to the students' activity and the results of the OATLP model in a specific cycle. Our aim is to highlight the interdisciplinary character of the research and the dialogue between theory and practice. Within EDR such a dialogue informs the development of OATLP. As an example of the OATLP, we have shown one complete implementation of the cycle.

4. Conclusions, Limitations and Directions for Future Directions

In this article, we presented the first results of the OPEN-MATH research project funded by the Free University of Bozen-Bolzano, whose aim is to develop inclusive mathematics communities of learners. We argue that the specific nature of both mathematical thinking and mathematical learning offers a privileged point of view on inclusion and the ensuing ethical and political issues that underpin education in general. We believe also that inclusion fosters reflections on ethical and political aspects that are molding recent philosophical reflections about Mathematics Education.

The aim of the project was to devise an activity model for inclusive mathematical thinking and learning. In order to accomplish our objective, we drew on the Theory of Objectification to frame Mathematical Learning and on Open Learning, within the strand of Inclusive Education, to frame inclusion. In this paper, we focused on the research path that led us to the design of a cycle of activities for inclusive mathematics teaching and learning, the OATLP.

In accordance with a dynamic understanding of theory, the paper described the construction of a conceptual framework that is the outcome of the profound dialogue between theory and practice carried out during the OPEN-MATH project. We have moved along the strand of the pragmatic bricolage for the purpose of understanding and implementing inclusive mathematics learning as an empirical phenomenon and constructing the model for inclusive mathematical activities.

With regard to theory, our study is characterized by the networking of the Theory of Objectification and Open Learning according to the combining-coordinating strategy, within the landscape of possible connecting strategies proposed by Prediger et al. [49]. Our concern was to respect the dialectics between social interaction and individual self-determination that characterizes our understanding of inclusion in mathematics, deriving from the encounter of the Theory of Objectification and Open Learning: mathematics learning as a process of objectification and inclusion as differentiation for all students. The combining-coordinating strategy, according to its defining features, allowed us to move along a pragmatic bricolage in order to define, understand and implement inclusive mathematics learning as an empirical phenomenon and the ensuing model of activity.

The outcome of the networking was a conceptual framework characterized by the following elements: the definition of inclusion for mathematics, the features of mathematical activity based on multi-modal sensuous cognition that intertwines both social interaction and individual self-determination, and the model for inclusive mathematical activities.

From the point of view of research methodology, we want to emphasize the collaborative aspect of EDR. In particular, dealing with problems in real context, which involves many actors from different levels (students, teachers, schools, institutions, etc.), requires an interdisciplinary approach that takes into account the different perspectives on the problem: the problem of inclusion, in particular, involves the different school's actors as well as researchers from different disciplines. The aim of working in the context of mathematics led to the construction of a two-fold disciplinary team, consisting of researchers in Mathematics Education and researchers in general education, with a particular interest in the inclusive paradigm. From the methodological point of view, this has brought us to the choice of EDR, and in particular to a re-discussion of the salient aspects of classroom practices and interactions observed in relation to inclusion in the mathematics classroom. This has resulted in a theoretical categorization for analyzing data that considers both purely disciplinary aspects, related to the learning of mathematics (like the categories referring to the levels of generalization and the use of artifact), and aspects related more closely to inclusion in his social aspects (the participation scale). In other words, the observation categories constructed for the classroom interventions and for the interviews were derived from a process of questioning different researchers' views on inclusion and learning, and from the selection of observable processes according to the two perspectives. This work was fundamental in order to work on categories that allow the improvement of the design that was gradually implemented both from the point of view of the mathematical objectives, which concern modes of communication, generalization, and use of artifacts, and from the point of view of participation in the activity and student self-determination, also from a relational point of view. The working strategy of EDR is effective in this process because it allows a fruitful relationship to be maintained between classroom implementation and theoretical understanding.

Moreover, the construction of the educational intervention OATLP is the embodiment of this theoretical and methodological work. Specifically, joint work has been done regarding the stations, with activities that are meaningful from a mathematical point of view and inclusive with respect to different cognitive styles, in a design that has involved researchers from the two areas. In the shown example, related to the Pythagorean theorem, the design of the researcher regarding the planning of the intervention has been made visible during all the phases of the cycle OATLP, in relation to the choice of the activities and to the role assignation, or with the definition of the help request presented in Section 3.3.

The present study has three limitations. The OATLP has been implemented with only one class of middle school students and further development of the cycle could be necessary with younger and older students. Our study has been carried out in a longitudinal and exhaustive study, but it involves only one group of students. Therefore, the research is at the present stage quantitatively not significant. The third macro-phase (evaluation-reflection) has not yet been performed, thereby limiting grounding of the OATLP model.

Such limitations open directions for future research, both from a theoretical and experimental point of view.

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